

Flood Routing Models for a Watercourse- A Review

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Abstract—Flood Routing is a mathematical procedure for predicting the changing magnitude, Speed and shape of a flood wave as a function of time at one or more points along a Watercourse. When a flood wave moves downstream a river, the wave configuration will be modified due to channel irregularities and roughness. Flood routing is important in the design of flood protection measures, to estimate how the proposed measures will affect the behavior of flood waves in rivers, so that adequate protection and economic solutions may be found. Recent developments in flood routing have resulted in several numerical models with different features. These models produce different results, and the inconsistency between computed and observed flows varies depending on the values of the channel's friction coefficient and bed slope. There are five flood routing models have been compared, the dynamic wave, the characteristic, the kinematic wave, Muskingum-Cunge and UBC Flow model. The main aim of this article is to find the most reliable model for a particular range of combinations of channel friction coefficient and bed slope.

1. INTRODUCTION

Flood is still one of the most important natural hazards threatening societies around the world and causes significant amount of damage [1]. Accurate estimation of this natural phenomenon and its propagation along river system can save thousands of people and a large amount of investment. Flood routing is the process of determining progressively the timing, shape, and amplitude of a flood wave as it moves downstream to successive points along the river. In general flood is unsteady, its mathematical description is nonlinear and there is no analytical solutions for numerous river engineering problems that can be conveniently investigate by means of mathematical models [2]. Mathematical models must properly describe the physical processes and provide a numerical solution to a system of differential equations that solved together with suitable boundary conditions and empirical relationships that describe resistance to flow and turbulence [3]. The differential equations, describing river problems are usually simplified forms of the equations conservation of mass and momentum, leading to a set of partial differential equations involving two independent variables (time and space or two spatial variables).

There are five flood routing models have been compared, the dynamic wave, the characteristic, the kinematic wave, Muskingum-Cunge and UBC Flow model. The first three models are based on a hydraulic approach, but they solve the unsteady flow equations with different methods. The fourth model is based on the hydrological approach, and the last one is a hybrid model [4]. With some modifications, these five different models can run using the same input parameters, and their results can be compared.

Once a river engineering problems have been defined and a mathematical model chosen, field data need to be gathered to describe initial and boundary conditions, geometrical similitude, materials properties and design condition.

2. FLOOD ROUTING MODELS

2.1 The dynamic wave models

2.2 The characteristic models

2.3 The kinematic wave models

2.4 Muskingum-Cunge models

2.5 UBC Flow model models

2.1 The Dynamic Wave Models

The following assumptions are used in the derivation of the governing equations: (1) the pressure distribution is hydrostatic, (2) the velocity is uniformly distributed over a channel section, (3) the average channel bed slope is small, (4) the flow is homogeneous and incompressible, and (5) there is no lateral flow [5]. Based on the assumptions, the continuity and momentum equations, the Saint-Venant equations, can be respectively expressed as follows:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0 \dots \dots \dots (1)$$

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{Q^2}{A} \right) + gA \left(\frac{\partial y}{\partial x} - S_0 \right) + gAS_f = 0 \dots (2)$$

Where,

Q= discharge,

A= cross-sectional area of flow,

x = horizontal coordinate along the channel,

t =time,

g = acceleration due to gravity,

y= flow depth,

S_o= slope of the bottom of the channel,

S_f = friction slope.

The equations above have two independent variables, x and t, and two dependent variables, the discharge Q and flow cross-sectional area A. When the Manning formula is used to represent the flow resistance, the friction slope is expressed as

$$S_f = \frac{n^2 V |V|}{R^{4/3}}$$

Where,

n = Manning’s roughness coefficient;

V = cross-sectional average flow velocity,

V=Q/A; and R is the hydraulic radius, and R = A/P, where P is the wetted perimeter. The term V |V| has the magnitude of V² and the sign is positive or negative depending on whether the flow is downstream or upstream, respectively.

If appropriate initial and boundary conditions are prescribed, the numerical solutions of Eqs. (1) and (2) can be obtained. Implicit finite difference schemes have been proven to be more efficient in the numerical treatment of the one-dimensional unsteady flow in rivers with a free surface than other methods such as the explicit and characteristic methods. For example, because of the numerical stability characteristics of the finite difference equations, theoretically, the implicit method does not restrict the size of the time step [6]. Larger values of time steps enable the implicit method to be more computationally efficient than other methods, particularly for long-duration floods.

2.2 The characteristic Model

In this method the partial differential equations of Saint-Venant converted to pair ordinary differential equations and finally solve by using finite difference scheme. This method helps to better understand the physics of shallow water [7]. Moreover the initial and boundary conditions in this way could be better obtained. The governing equations of Characteristics Model are shown as below:

$$\left[\frac{dV}{dt} \right] + \frac{g}{c} \left(\frac{dy}{dt} \right) = g(S_0 - S_f) \frac{dx}{dt} = V + C.. (3)$$

$$\left[\frac{dV}{dt} \right] - \frac{g}{c} \left(\frac{dy}{dt} \right) = g(S_0 - S_f) \frac{dx}{dt} = V - C ... (4)$$

This model is derived from kinematic wave theory. In this model routing coefficients are determined by using graphical scheme. If appropriate initial and boundary conditions are prescribed, the numerical solutions of Eqs. (3) and (4) can be obtained.

As we mentioned before flood routing bases on two partial differential equations which named Dynamic Wave Equations and contain Mass continuity and Momentum equation:

$$T \frac{\partial y}{\partial t} + A \frac{\partial v}{\partial x} + V \frac{\partial A}{\partial x} = 0..... (5)$$

$$\frac{\partial v}{\partial t} + V \frac{\partial v}{\partial x} + g \frac{\partial y}{\partial x} = g (S_0 - S_f)..... (6)$$

Because of water viscosity property, flood movement in open channels occurs continually. Consequently, in each cross section through the channel, initial and boundary discharge conditions impress flood condition in next time steps. On the other hand, most of the numerical methods (finite difference and finite element methods) which are used to solve Dynamic Wave Equations for flood routing description contain schemes with two or three time [7]. Furthermore, based on Courant Equation, there is some limitation in range of Δt and Δx definition. To wrap it up, it could be said that time range of input statistics definition; and also, sensitivity of each case study flood routing modeling to input discharge statistics have to be calculated [6].

2.3 The Kinematic Wave Model

The kinematic wave method has been used to solve the unsteady flow equations in many flood routing models. The kinematic wave approach for overland flow as well as for stream flow in a watershed [8]. Some of reasons for using this method are because it is simple, it does not require downstream boundary conditions for solving the above equations, and it was believed that its approach approximates the natural condition of flood flow. The method assumes that the effects of the inertia and depth slope terms in natural flood flow are small compared with the bed slope term, so that they can be neglected [2]. The friction term in momentum equation mainly depends on bed slope.

Conservation of mass and momentum leads to the continuity equation:

$$\frac{\partial y}{\partial t} + \frac{A}{T} \left(\frac{\partial u}{\partial x} \right) + u \left(\frac{\partial y}{\partial x} \right) - \frac{q}{T} = 0 (7)$$

and the momentum equation:

$$\frac{\partial y}{\partial t} + u \left(\frac{\partial u}{\partial x} \right) + g \left[\frac{\partial y}{\partial x} + (S_f - S_0) \right] + \frac{qu}{A} = 0(8)$$

Where,

y = depth of flow

u = velocity of flow

t = time

x = distance, through the longitudinal axis of the channel
 A = wetted cross-sectional area
 T = top width of water in the channel
 S_f = friction slope, depends on channel's geometry, friction, and discharge
 S_o = bed slope = $\frac{\partial x}{\partial y}$
 q = lateral inflow
 g = acceleration due to gravity
 Assume, S_f = S_o

With this assumption, it is much simpler to find the solution to the unsteady flow equations. Eq. (8) leads to a condition of steady uniform flow in which Manning's equation can be applied.

$$Q = \frac{1}{n} AR^{2/3} S_o^{1/2} \dots\dots\dots (9)$$

Eq. (9) can also be expressed as, $Q = \alpha A^b$ or $A = \beta Q^a \dots\dots\dots (10)$

Comparing eq. (9) with eq. (10): $\beta = \frac{nP^{2/3}}{S_o^{1/2}}$, and a = 0.6. And using, so variable Q (discharge) and A (area) to substitute y (depth) and u (velocity), eq. (10) will appear as

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = Q \dots\dots\dots (11)$$

Differentiating eq (10) to t gives,

$$\frac{\partial A}{\partial t} = a\beta Q^{a-1} \frac{\partial Q}{\partial t} \dots\dots\dots (12)$$

and substituting eq. (11) to eq. (12) gives,

$$a\beta Q^{a-1} \frac{\partial Q}{\partial t} + \frac{\partial Q}{\partial x} = Q \dots\dots\dots (13)$$

Using the total derivative for increment in discharge flow:

$$dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial t} dt \dots\dots\dots (14)$$

With $q = dQ/dx =$ lateral inflow per unit length, eq.(13) and eq.(14) result:

$$\frac{dy}{dt} = \frac{1}{a\beta Q^{a-1}} \dots\dots\dots (15)$$

From differentiating eq.2.11 and rearranging it, gives:

$$\frac{dQ}{dA} = \frac{1}{a\beta Q^{a-1}} \dots\dots\dots (16)$$

Eq.(15) and(16) give $C_k = dx/dt = dQ/dA$, where $C_k =$ kinematic wave velocity. An observer moving along the river bank at velocity $C_k = dx/dt$ would see the flow rate increasing at the rate of $dQ/dx = q$, and if $q = 0$, the observer would see a constant discharge. The method is most useful in a quick responding (urban) watershed, where lateral inflow and short channels prevent the formation of the kinematic shock.

2.4 Muskingum-Cunge Models

One of standard methods of hydrological flood routing models is the well-known Muskingum model. The model is based on conservation of mass which applied to storage, inflow and outflow within the reach.

Storage equation:

$$S = K(Ix + (1 - x)O) \dots\dots\dots (17)$$

Continuity equation:

$$\frac{ds}{dt} = I - O \dots\dots\dots (18)$$

Where,

s = absolute storage within the reach

i = inflow discharge

o = outflow discharge

x = a weighting factor

K = a gradient of the storage vs. the weighted flow curve, and K is related to time lag or travel time of the flood wave through the reach [9].

From the equations above, it seems that the Muskingum method is really a simple method. The difficulties are only how to determine proper values of K and x which lead to an accurate result or prediction. K and x could be determined graphically from the values of weighted flows vs. their pertinent storages. However, all models mentioned above require a large amount of historical data on storage, inflow and outflow within the reach. It means that each particular reach (with particular S_o and Manning's n) should have its historical data set. And for other conditions of S_o or Manning's n, it always needs another historical data set [8].

Cunge found that parameter K and x can be obtained from the length of the reach (Δx), the flood wave celerity (c), and the discharge per unit width (Q_w), instead of determining them from a large amount of historical data. Later, Ponce and Yevjevich continued this development by assuming that K and x are variable parameters. K and x are allowed to vary in time and space (distance) following the fluctuation of the flow. Flow depth for each point grid is calculated by iteration. Knowing the flow depth, one can find c and Q_w for each distance step using a three-point average of the values at grid points [9]. From these values, one can calculate Muskingum's parameters. These parameters vary in time and space during the calculation of determining them from a large amount of historical data.

2.5 UBC Flow models

The University of British Columbia (UBC) Flow model was originally developed to cope with local problems encountered (i.e., ungagged lateral inflow coming from snowmelt, steep bed slope, short travel time, etc.) in the Fraser River, British

Columbia, Canada. The method has been tested over many years on the Fraser river system, and on other rivers (e.g., North Saskatchewan River, the upper reaches of Columbia river) [2]. The results indicated that the flow model is flexible, easy to calibrate, and suitable for both large and small rivers. It is also easy to modify the storage portion of the model for lake and reservoir routing. Lateral inflow is excluded from the routing calculation within the reach, because it can be added in at downstream boundary. Routing coefficients are determined directly from velocity-discharge and area discharge relationships [10].

UBC Flow uses a kinematic wave approach, and then adds an extra term to account for the influence of channel storage. This extra term is the depth slope term, $\frac{\partial y}{\partial x}$. Instead of using a finite difference approximation, the solutions are determined by decomposing simply into a pure of translation of the flood wave, followed by a decay function. These two separate operations involve calculating the travel time from kinematic wave velocity, and then by routing the flow through a simple linear reservoir [11]. The UBC Flow model consists of two independent procedures. The first procedure is a calculation of the travel time for a particular reach as function of river stage (e.g., velocity-discharge and area-discharge relationship). The inflow is translated through the reach using this travel time to give translated outflow [6]. The second procedure is routing. The translated outflow is routed through a simple linear reservoir which represents the storage characteristic of the reach. Lateral inflow coming from snow melt or any additional tributary inflow can be added at the end of each reach.

These two simple and independent procedures mentioned above make the UBC Flow model flexible and easy to fit to a real system. Also, the physical behaviour of a flood wave in a channel can be almost completely described by a travel time and the subsidence of the flood wave (modeled by routing through a reservoir) [2, 11]. The travel times and reservoir storage are related to the channel's parameters (i.e., S_0 , Manning's n , bottom width, bank slope) as velocity-discharge and area-discharge relationships. This condition makes the model accurate and comparable to other methods that use basic river flow data.

3. COMPARISON AND CONCLUSION

The first three models are the dynamic wave, the characteristic and the kinematic wave model. They are based on a hydraulic approach; however, they use different methods to solve the non-linear partial differential equations of unsteady flows. The fourth model is Muskingum-Cunge model based on a hydrological approach, and the last one is a hybrid model that was developed by the University of British Columbia. Since the above models use different ways to obtain their results, they can produce different quality of results.

The dynamic wave method, mathematically the most complicated one among them. It can obtain the results by

solving simultaneously the complete unsteady flow equations throughout the reach. The characteristic method simplifies the equations through characteristic forms, while the kinematic wave method eliminates the inertia and depth slope terms from the equations. The UBC Flow and Muskingum-Cunge methods use different equations that are based on the hydrological approach. Since the five models have different features, originally they need different input parameters. They are also having different boundary conditions. With some modifications, however, it is possible to compare them for the purpose of this study.

The inertia and the depth slope terms of the unsteady flow equations influenced not only the way of solving the equations, but also the results. By neglecting them, to solve the equations becomes much easier. However, the results can be greatly affected, especially for a channel with big value of friction coefficient (e.g., covered by dense vegetation, rough bed) or with nearly horizontal bed slope. The flood routing models omitting the inertia and the depth slope terms are simpler than those using the complete unsteady equations. By using the complete unsteady flow equations, the dynamic wave model has a complex feature, but, this model is flexible, applicable, and reliable.

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